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Towards an improvement of the geoid model in Japan by GOCE data: A case study of the Shikoku area

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The performance of the recently-released global geopotential models (GGMs) based on 2, 8 and 12 months of data collected by the Gravity field and steady-state Ocean Circulation Explorer (GOCE) is evaluated using geoid undulations and free-air gravity anomalies over Japan. Comparisons over the four main islands reveal that EGM2008 performs better than GOCE and related GGMs in Hokkaido, Honshu and Kyushu. However, GOCE and related GGMs perform better than EGM2008 in Shikoku. GOCO02S, GOCE-DIR3 and GOCE-TIM3 have a similar performance, and the best, in Shikoku. Given that GOCE-TIM3 relies exclusively on GOCE data, it is assessed further for geoid determination in Shikoku. To evaluate the actual improvement of the geoid model in the Shikoku area by GOCE-TIM3, the geoid over Shikoku is determined from EGM2008 and a combination of GOCE-TIM3 with EGM2008. There is an improvement in the standard deviation from ±8.7 cm, when EGM2008 is used, to ±6.6 cm, when GOCE-TM3/EGM2008 is used. The first improvement of the geoid model over Japan by GOCE data is evident in Shikoku.

Key words: Geoid model, gravity, GOCE, EGM2008.

1. Introduction

Geoid modelling over Japan remains a challenge, especially with respect to the establishment of a consistent vertical datum. The last decade has seen a concerted effort towards the realisation of a precise geoid model over Japan (e.g. Kuroishi *et al*., 2002; Kuroishi and Keller, 2005; Kuroishi, 2009; Odera *et al*., 2012).

Several global geopotential models (GGMs), both combined and satellite only, exist today. The evaluation of the performance of GGMs is necessary for the selection of an optimal model for geoid determination. Some of the GGMs that have been used for geoid modelling in Japan include OSU91A (Rapp *et al*., 1991), EGM96 (Lemoine *et al*., 1997), GGM02C (Tapley *et al*., 2005) and EGM2008 (Pavlis *et al*., 2008). However, EGM2008 performs better than the other mentioned GGMs over Japan.

Recently, a number of GGMs based on the data collected by the Gravity field and steady-state Ocean Circulation Explorer (GOCE) have been released. Some of the evaluations of GOCE GGMs can be found in Janák and Pitoňák (2011), Hirt *et al*. (2011) and Gruber *et al*. (2011). We evaluate the performance of the recently-released GGMs based on 2, 8 and 12 months of data collected by GOCE using geoid undulations and free-air gravity anomalies over Japan. Further evaluations are carried out over each of the four main islands. However, Honshu is divided into three parts (north, central and west) because of its size and geometry. This kind of sub-regional evaluation of GGMs is being tested over Japan for the first time. The evaluated GOCE and related GGMs include GOCE-DIR1, 2, 3 (Bruinsma *et al*., 2010; Pail *et al*., 2011), GOCE-TIM1, 2, 3 (Pail *et al*., 2010b, 2011), GOCE-SPW1, 2 (Migliaccio *et al*., 2011) and GOCO01S, 02S (Pail *et al*., 2010a; Goiginger *et al*., 2011). From the preliminary evaluations over Japan, it is found that GOCE and related GGMs can improve the geoid model over Shikoku.

To determine the actual improvement, two geoid models are computed over Shikoku using EGM2008 and GOCE-TIM3/EGM2008. In both cases, the same terrestrial gravity data sets are used. The Stokes-Helmert scheme in a modified form is applied for the determination of the geoid, using an empirically-determined optimal spherical cap-size, and Kriging is used for gridding the residual gravity anomalies. The standard deviation of the differences between gravimetric and GPS/levelling geoid undulations is used to assess the two geoid models. The results of the evaluations are presented. The paper concludes with a comparison of the derived gravimetric and GPS/levelling geoid undulations over Shikoku.

2. Evaluation of GGMs

The distribution of GPS/levelling and first-order gravity data over the four main islands is given in Fig. 1. The number of GPS/levelling and first-order gravity data in the six sub-regions is given in Table 1. A preliminary evaluation of GGMs based on 2 and 8 months of GOCE data over Japan shows improvement by GOCE-release 2 (8 months data) compared to GOCE-release 1 (2 months data) in Japan. However, GOCE-DIR1 performs better than GOCE-DIR2 over Japan. Similar results have been observed over Central Europe (Janák and Pitoňák, 2011). Therefore, only re-

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Table 1. Number of GPS/levelling and first-order gravity data points in six sub-regions of Japan.

Data	Hokkaido	North Honshu	Central Honshu	West Honshu	Shikoku	Kyushu	Whole
GPS/lev	163	171	163	158	56	105	816
Gravity	1,431	1,368	1,620	1,166	401	965	6,951

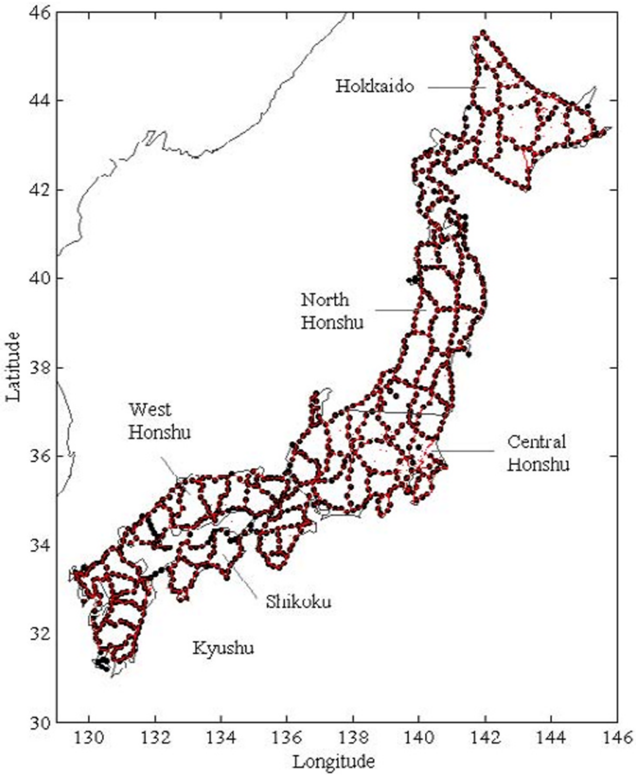

Fig. 1. Distribution of GPS/levelling (big black dots) and first-order gravity (small red dots) data over the four main islands.

leases 2 and 3 of GOCE-related GGMs and GOCE-DIR1 are considered for detailed evaluation. The standard deviations of the differences between GPS/levelling, and GGMs-implied, geoid undulations are given in Table 2, while the standard deviations of the differences between the observed, and the GGMs-implied, free-air gravity anomalies are given in Table 3. In these tables, all the models are truncated to 150, 180, 210 and 240 degrees.

In summary, the performance of EGM2008 and GOCE-related GGMs over Japan is practically the same at 150 degrees, although EGM2008 performs better at the spectral bands 180, 210 and 240 degrees. GOCE-TIM3 performs better than GOCE-TIM2 over Japan. Although GOCE-DIR3 performs better than GOCE-DIR2, it still performs slightly below GOCE-DIR1 over Japan. It only offers an improvement at 180 degrees, but the accuracy degenerates at the higher degrees. It should be noted that GOCE-DIR1, 2 and 3 are different in terms of the background data sets involved. Hence, the results of the comparisons over Japan are not so strange.

The comparisons over the four main islands reveal that EGM2008 performs better than GOCE, and related GGMs, in Hokkaido, Honshu and Kyushu. The good performance of EGM2008 over most parts of Japan may be attributed to the inclusion of terrestrial gravity data in the development of EGM2008. However, GOCE, and related GGMs, perform better than EGM2008 in Shikoku. We therefore suspect errors in the gravity data included in EGM2008 from the Shikoku area. GOCE-TIM3, GOCO02S and GOCE-DIR3 have a similar, and better, performance in Shikoku. Given that GOCE-TIM3 relies exclusively on GOCE data, it is considered for geoid determination in Shikoku.

3. Geoid Determination over Shikoku

To evaluate the actual improvement of the geoid model in the Shikoku area by GOCE-TIM3, the geoid in Shikoku is determined from EGM2008 (up to 2, 190 degrees) and a combination of GOCE-TIM3 (up to 180 degrees) with EGM2008 (from 181 to 2, 190 degrees), that is, GOCE-TIM3/EGM2008. Stokes's integral formula for geoid determination (Stokes, 1849; Heiskanen and Moritz, 1967) has been used. A modified Stokes's formula, excluding small errors due to the ellipsoidal effects, is given as,

$$N = N_{\mathrm{GGM}} + \frac{R}{4\pi\gamma}\iint_{\sigma_o} \Delta g^r S^{\mathrm{ME}}(\psi)\,d\sigma + N_{\mathrm{ind}}, \qquad (1)$$

where N is the gravimetric geoid undulation, N_{GGM} is the geoid undulation obtained from EGM2008 or GOCE-TIM3/EGM2008 after applying the zero-degree term (with respect to GRS80), Δg^r is the residual gravity anomaly,

Table 2. Standard deviations of the differences between GPS/levelling and GGMs-implied geoid undulations in Japan (units in cm), n represents the spherical harmonic degrees.

GGM	Hokkaido	North Honshu	Central Honshu	West Honshu	Shikoku	Kyushu	Whole
$n = 150$							
EGM2008	101.3	66.3	98.2	93.0	73.5	43.3	88.4
DIR1	101.1	67.7	98.7	90.8	72.0	42.4	88.0
DIR2	101.5	66.5	99.5	91.7	71.2	43.3	88.4
DIR3	101.4	66.4	99.6	92.2	71.2	43.5	88.5
SPWS 2	101.2	66.3	99.1	91.3	71.1	43.6	88.0
TIM2	101.6	66.3	99.3	91.6	71.1	43.6	88.3
TIM3	101.4	66.1	99.5	92.3	70.8	43.6	88.4
GOCO02S	101.3	66.1	99.1	92.2	70.8	43.3	88.3
$n = 180$							
EGM2008	75.2	53.6	60.9	71.3	61.0	43.2	64.7
DIR1	75.6	55.7	61.3	72.0	60.0	45.2	65.4
DIR2	77.7	53.7	63.0	70.5	59.4	45.8	65.7
DIR3	77.2	53.9	62.3	70.3	57.6	43.6	65.1
SPWS 2	77.7	55.0	63.1	70.1	59.0	45.0	65.8
TIM2	77.8	53.8	62.7	70.3	59.6	45.5	65.7
TIM3	76.9	54.1	62.7	70.5	58.1	44.3	65.4
GOCO02S	77.3	54.0	62.9	70.3	58.6	45.7	65.6
$n = 210$							
EGM2008	63.9	51.6	55.3	52.7	43.2	39.1	54.7
DIR1	64.0	53.1	56.4	54.2	42.7	42.2	55.8
DIR2	67.0	52.0	58.3	53.8	41.6	42.8	56.6
DIR3	66.4	53.7	56.5	53.0	41.8	39.3	56.0
SPWS 2	68.0	53.1	57.5	57.2	43.9	39.7	57.6
TIM2	68.2	52.3	57.4	52.9	42.3	41.7	56.6
TIM3	66.5	52.7	56.2	52.9	42.1	40.2	55.9
GOCO02S	67.8	52.3	57.7	53.4	42.0	41.8	56.6
$n = 240$							
EGM2008	56.4	42.6	54.2	42.9	41.5	31.0	48.4
DIR1	56.6	45.7	55.9	45.3	42.6	32.8	50.0
DIR2	65.3	41.0	59.7	45.3	41.8	43.1	52.9
DIR3	61.2	44.3	56.6	48.4	41.2	34.7	51.6
SPWS 2	67.1	49.6	57.4	54.4	42.6	38.7	56.1
TIM2	63.9	42.8	59.1	45.6	40.9	38.6	52.5
TIM3	60.1	43.0	57.8	47.2	41.9	34.8	51.3
GOCO02S	63.6	43.0	59.3	46.3	40.7	38.8	52.6

N_{ind} is the indirect effect on the geoid due to gravity reduction, and $S^{\text{ME}}(\psi)$ is the Meissl's modified kernel (Meissl, 1971).

A description of gravity data used in this study can be found in Odera *et al*. (2012). The direct terrain effects (DTE), and the primary indirect terrain effects (PITE), are computed by the integral formulae proposed by Martinec and Vaníček (1994a, b) using a 50-m digital elevation model. The Kriging technique (Krige, 1951) is used for gridding residual gravity anomalies on a 1 by 1.5 arc-minute grid. A spherical cap-size of 40 km is adopted for the computations, after empirical evaluations. It should be noted that the classical Moritz formula (Moritz, 1980) and a planar formula (Wichiencharoen, 1982) are used for the computation of DTE and PITE, respectively, in the previous geoid model for Japan (Odera *et al*., 2012).

The comparisons are carried out using 56 GPS/levelling points in Shikoku. Let the two geoid models developed using EGM2008 and GOCE-TIM3/EGM2008, incorporating the contribution of the local gravity data, be referred to as geoid models A and B, respectively. Figure 2 shows the differences between the gravimetric geoid (A) and the GPS/levelling geoid undulations, while Fig. 3 represents the differences between the gravimetric geoid (B) and the GPS/levelling geoid undulations in Shikoku area. The statistics of the differences between gravimetric and GPS/levelling geoid undulations in Shikoku for the two geoid models are given in Table 4.

It can be seen from Table 4, that GOCE is already capable of improving the geoid model in the Shikoku area after 12 months of observations. There is an improvement in the standard deviation from ± 8.65 cm (for geoid model A) to ± 6.56 cm (for geoid model B), representing an improvement of 24.2%. A similar comparison using the previous geoid model for Japan (Odera *et al*., 2012) gives a standard deviation of ± 8.69 cm over Shikoku. This means that the

Table 3. Standard deviations of the differences between observed and GGMs-implied free-air gravity anomalies in Japan (units in mGal), n represents the spherical harmonic degrees.

GGM	Hokkaido	North Honshu	Central Honshu	West Honshu	Shikoku	Kyushu	Whole
$n = 150$							
EGM2008	41.6	27.0	40.2	33.5	27.8	18.4	35.7
DIR1	41.5	27.4	40.4	33.1	27.5	18.5	35.7
DIR2	41.6	27.2	40.4	33.2	27.5	18.4	35.7
DIR3	41.6	27.2	40.4	33.3	27.6	18.3	35.7
SPWS 2	41.6	27.2	40.4	33.1	27.5	18.4	35.7
TIM2	41.6	27.1	40.4	33.2	27.5	18.4	35.7
TIM3	41.6	27.1	40.4	33.3	27.6	18.3	35.7
GOCO02S	41.5	27.1	40.5	33.4	27.5	18.3	35.7
$n = 180$							
EGM2008	37.1	26.1	40.1	28.5	24.1	21.4	33.0
DIR1	37.3	26.3	40.2	28.7	23.6	21.8	33.2
DIR2	37.7	26.2	40.3	28.3	24.0	21.8	33.3
DIR3	37.6	26.2	40.3	28.4	24.0	21.5	33.2
SPWS 2	37.7	26.3	40.3	28.4	23.8	21.5	33.3
TIM2	37.8	26.2	40.4	28.3	24.0	21.8	33.3
TIM3	37.6	26.2	40.3	28.4	24.1	21.6	33.2
GOCO02S	37.6	26.2	40.5	28.3	24.1	21.8	33.3
$n = 210$							
EGM2008	34.5	27.4	41.9	25.7	23.5	22.0	32.5
DIR1	34.6	27.7	42.3	26.2	23.3	22.6	32.8
DIR2	35.4	27.8	42.4	26.1	22.2	21.8	32.9
DIR3	35.1	28.0	42.0	25.7	23.0	21.7	32.7
SPWS 2	35.4	27.3	41.8	26.2	23.1	21.0	32.7
TIM2	35.6	27.8	42.3	25.7	22.7	21.7	32.9
TIM3	35.0	27.8	41.9	25.7	23.1	21.8	32.7
GOCO02S	35.4	27.7	42.4	25.8	22.9	21.7	32.9
$n = 240$							
EGM2008	32.5	26.1	42.1	23.3	26.2	20.5	31.4
DIR1	32.7	26.8	42.4	24.2	26.7	20.9	31.9
DIR2	35.1	25.9	43.4	24.8	24.5	21.6	32.6
DIR3	34.0	26.5	42.6	25.0	24.3	20.8	32.2
SPWS 2	35.3	26.6	41.6	25.7	22.9	21.0	32.4
TIM2	34.6	25.8	43.0	24.7	23.8	20.8	32.3
TIM3	33.6	26.1	42.8	25.0	24.3	20.6	32.1
GOCO02S	34.5	25.9	43.1	24.9	23.9	20.8	32.3

Table 4. Statistics of the differences between gravimetric and GPS/levelling geoid undulations in Shikoku for geoid models A and B (units in cm).

Geoid model	Minimum	Maximum	Mean	SD
A	−15.71	30.16	−1.34	8.65
B	−13.56	26.81	−0.32	6.56

use of integral formulae for computing DTE and PITE gives a very slight improvement in the geoid model over Shikoku, probably due to the low elevation (less than 2,000 m).

The differences between gravimetric and the GPS/levelling geoid undulations are smoother for geoid model B than geoid model A, as shown in Fig. 3 and Fig. 2, respectively. We suspect that the relatively low performance of EGM2008 in Shikoku is partly due to errors in the terrestrial gravity data, included in the development of EGM2008, from the Shikoku area.

4. Conclusions

The performance of the recently-released GOCE and related GGMs is evaluated over Japan as a whole, and in the sub-regions, using GPS/levelling geoid undulations and free-air gravity anomalies. The performance of EGM2008 and GOCE-related GGMs over Japan is comparable, although EGM2008 generally performs better than the GOCE-related GGMs over Japan. It is noted that the performance of EGM2008 and GOCE-related GGMs is practically the same over Japan at 150 degrees. Comparisons over the four main Japanese islands show that GOCE and related GGMs perform better than EGM2008 in the

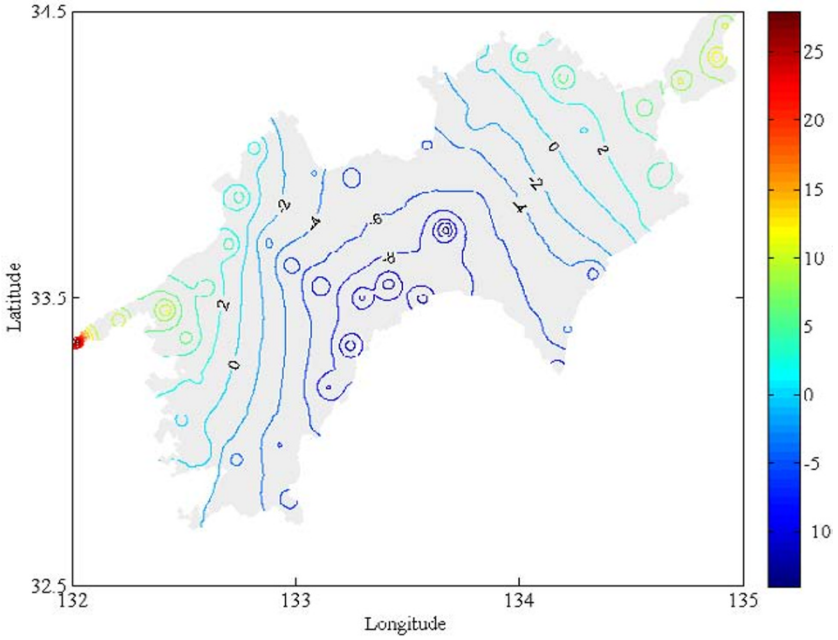

Fig. 2. Differences between the gravimetric geoid (A) and GPS/levelling geoid undulations in Shikoku (units in cm).

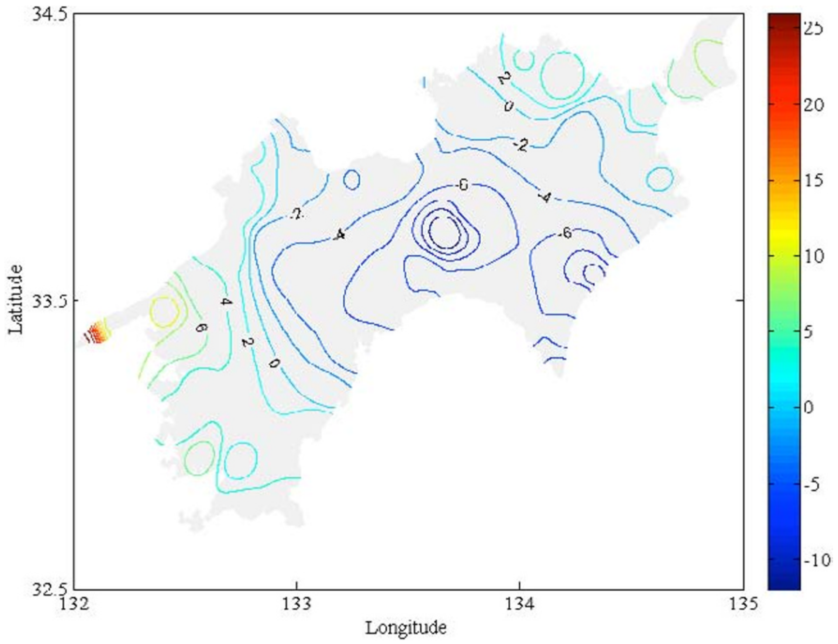

Fig. 3. Differences between the gravimetric geoid (B) and GPS/levelling geoid undulations in Shikoku (units in cm).

Shikoku area.

Two gravimetric geoid models on a 1 by 1.5 arc-minute grid covering the Shikoku area are developed from EGM2008 and GOCE-TIM3/EGM2008, incorporating the contribution of the local gravity data. There is an improvement in the standard deviation from ± 8.7 cm, when EGM2008 is used, to ± 6.6 cm, when GOCE-TM3/EGM2008 is used. There are good prospects for the improvement of the geoid model over Japan by GOCE data at the end of the mission. The improved geoid model(s) from the GOCE data will contribute to efforts towards the unification of vertical datums at national, regional and global levels.

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